7.7 – Higher Degree Polynomials

Daily Objectives:
1. Investigate extreme values and the end behavior of polynomial functions.
2. Identify possible degrees of a polynomial function by looking at its graph.
3. Identify and find the lowest-degree polynomial that has given roots.
4. Find additional roots when given one of more complex roots.
5. Relate the graphs of polynomial equations to the number and types of roots.

Local Maximum: A value of a function or graph that is greater than other nearby values.

Local Minimum: A value of a function or graph that is less than other nearby values.

End Behavior: The behavior of a function $y=f(x)$ for x-values that are really big or really small

Extreme Values: Another term for local maxima and minima

Example 1: Below the function $f(x)=x^3+2x^2-5x-6$ is graphed. Describe the following pieces of information:

a. End Behavior

As the x-values get larger, the y-values get larger.

As the x-values get smaller, the y-values get smaller

b. Extreme Values

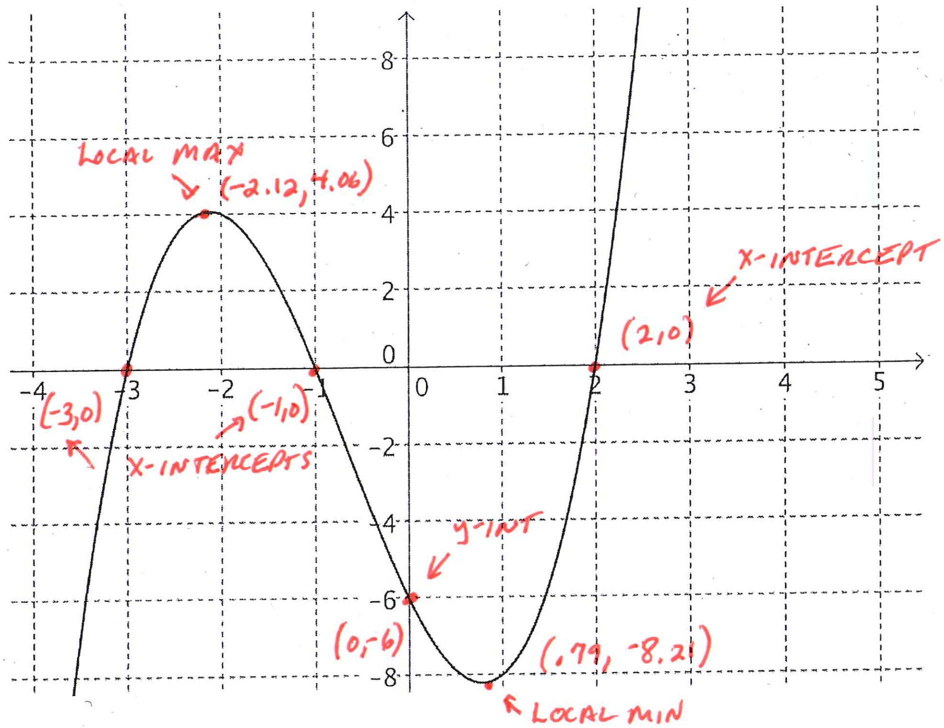
LOCAL MAX: (-2.12, 4.06)

LOCAL MIN: (.79, -8.21)

c. Intercepts

X-INT: (-3,0) (-1,0) (2,0)

Y-INT: (0,-6)



Example 2: Find a polynomial function whose graph has x-intercepts 3, 5, and -4, and y-intercept 180.

a. Factored From:

$y = a(x-3)(x-5)(x+4)$

$180 = a(-3)(-5)(4)$

$\frac{180}{60} = \frac{60a}{60}$

$3 = a$

$\boxed{y = 3(x-3)(x-5)(x+4)}$

b. General Form:

$y = 3[(x^2-5x-3x+15)(x+4)]$

$y = 3[(x^2-8x+15)(x+4)]$

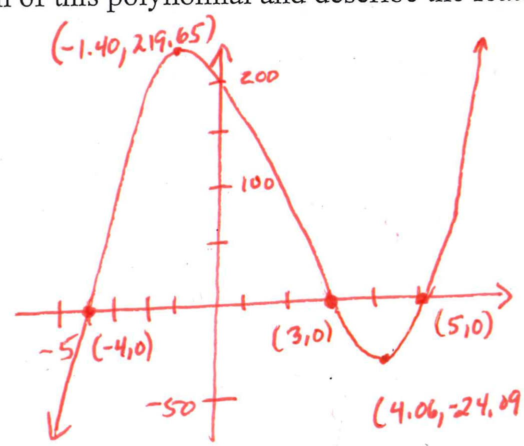
$= 3[x^3+4x^2-8x^2-32x+15x+60]$

$= 3[x^3-4x^2-17x+60]$

$\boxed{y = 3x^3-12x^2-51x+180}$

Sketch a graph of this polynomial and describe the features of its graph.

y MIN: 50
y MAX: 250



Example 3: Write a polynomial function with real coefficients and zeros x = 2, x = -5, and x = 3.

Factored Form:

$y=(x-2)(x+5)(x-3)$

General Form:

$y=(x^2+5x-2x-10)(x-3)$

$=(x^2+3x-10)(x-3)$

$=x^3-3x^2+3x^2-9x-10x+30$

$y=x^3-19x+30$

Example 4: Write a polynomial function with real coefficients that has a double root at 4.

Factored Form:

$y=(x-4)(x-4)$

or

$y=(x-4)^2$

General Form:

$y=(x-4)(x-4)$

$y=x^2-4x-4x+16$

$y=x^2-8x+16$

Example 5: Write a polynomial function with real coefficients that has a double root at -1 and a root at 1.

Factored Form:

$y=(x+1)(x+1)(x-1)$

or

$y=(x+1)^2(x-1)$

General Form:

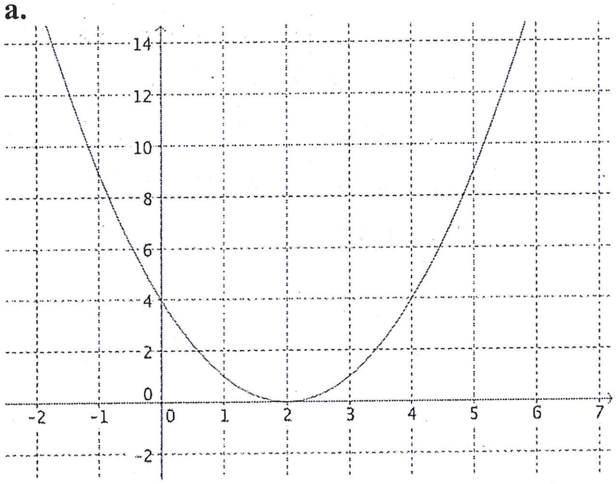
$y=(x^2+2x+1)(x-1)$

$=x^3+2x^2+x-x^2-2x-1$

$y=x^3+x^2-x-1$

Example 6: Write the equation for the polynomials graphed below in factored form.

a.



$y = a(x-2)^2$

$y = a(x-2)(x-2)$

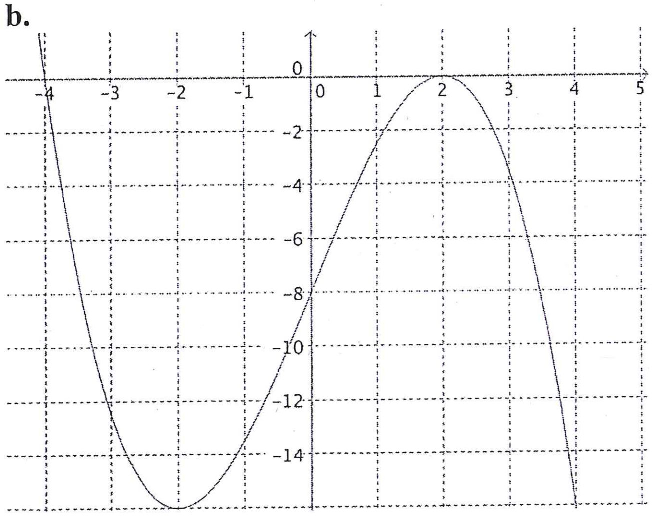
$4 = a(0-2)(0-2)$

$\frac{4}{4} = \frac{4a}{4}$

$a = 1$

$y = (x-2)^2$

b.



$y = a(x+4)(x-2)(x-2)$

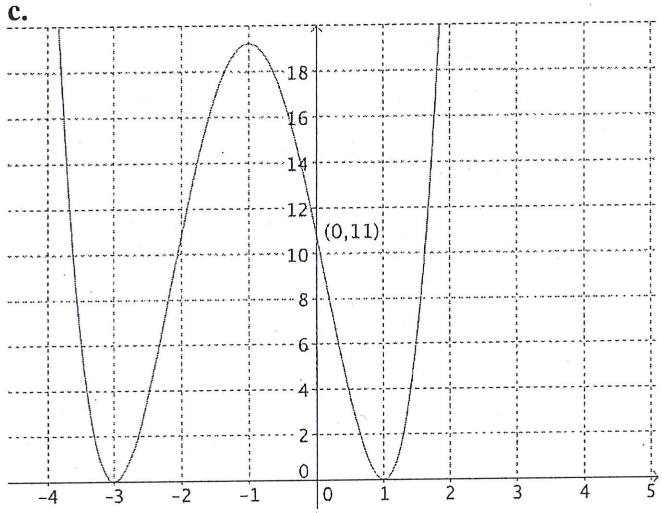
$-8 = a(4)(-2)(-2)$

$\frac{-8}{16} = \frac{16a}{16}$

$-\frac{1}{2} = a$

$y = -\frac{1}{2}(x+4)(x-2)^2$

c.



$y = a(x+3)^2(x-1)^2$

$y = a(x+3)(x+3)(x-1)(x-1)$

$11 = a(9)(1)$

$\frac{11}{9} = \frac{9a}{9}$

$\frac{11}{9} = a$

$y = \frac{11}{9}(x+3)^2(x-1)^2$